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A FAST NUMERICAL ASSESSMENT OF RAILWAY-INDUCED GROUND VIBRATION IN URBAN CONDITIONS

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This work presents a fast computational approach to quantify the ground vibration generated by the passing of urban railway vehicle over localized defects (e.g. rail joint or turnout). The methodology is based on a two-step simulation where the vehicle/track/foundation and track/soil dynamics are calculated separately. The vehicle/track interaction, providing the excitation source, is modelled by means of a coupled multibody/finite element model. The contact forces generated by the dynamic interaction of the wheels and the localized defects are then saved for use within the track/soil model. The latter is defined using a two-and-half dimensional (2.5D) approach based on a Fourier transform of the coordinate in the longitudinal direction of the track. To avoid restrictions imposed by this approach (geometry and dynamic characteristics invariant along the track direction), transfer mobility functions and wheel/rail force densities are used. The model is validated using both field (measurements on the Brussels tram network) and numerical methods (a 3D vehicle/track/soil model). Good agreement is observed, showing the potentiality of the proposed method in terms of computational efficiency.

Keywords: urban area, rail joint, ground vibration, scoping approach, 2.5D BE-FE model

1. Introduction

Predicting ground vibration generated by the passing of trains represents an important tool for vehicle and network designers and several approaches exist and offer different ways to evaluate mitigation measures [1]. According to the network type (high-speed lines, metro lines, urban lines), some hypotheses and limitations can be adopted:

- If an uniform excitation along the railway corridor acts during the passing of a train and if an invariant property is assumed along this longitudinal direction, the track could be considered as a periodic structure [2]; the reaction forces between the track and the subgrade can be deemed as identical but a certain time delay. This is the case of high speed lines with continuous welded rails or the case of periodic excitations [3].
- In the case of localized defects, the invariant hypothesis is no longer valid; the reaction forces between the track and the subgrade are important in the vicinity of the defect and decrease with distance from the source (Figure 1).

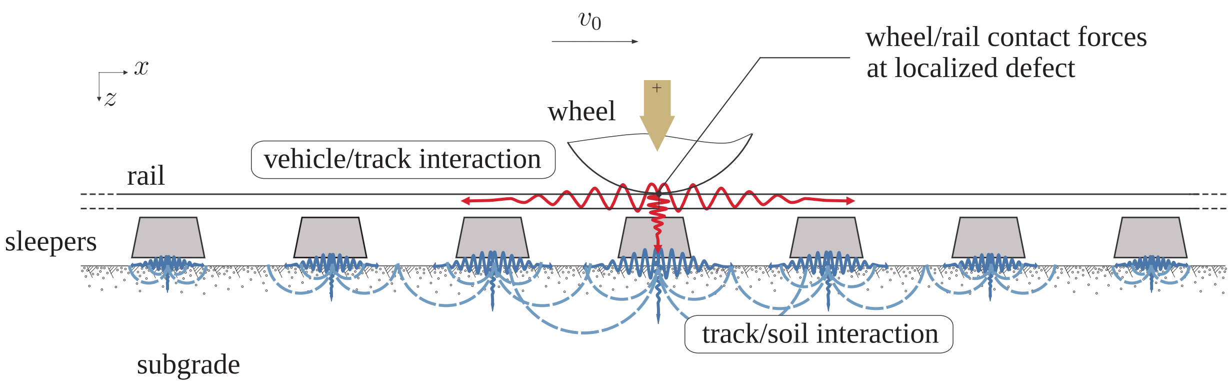


Figure 1: Wheel/rail and sleeper/subgrade interactions.

Based on this consideration, fast or scoping approaches were developed (e.g. [4]), avoiding long computational time needed by full 3D models [5, 6]. For example, to include realistic track irregularities into vehicle/track dynamic simulation model, Xu et al. [7] proposed a probability density evolution method to solve the transmission issue between track random irregularities and dynamic responses, improving the computational efficiency. However, the necessity of full complete approaches has been pointed out for particular problems. For example, the environmental effects of ground-borne vibrations generated due to localised railway defects was studied with a time domain, three-dimensional ground vibration prediction model [8]. This simulation approach was time-consuming but provided an in-depth sensitivity analysis of defect type and size on the vibration level.

More recently, Kouroussis et al. [9, 10] proposed a hybrid numerical/experimental assessment dedicated to the urban area to study such defects using the calculation of wheel/rail forces coupled to experimental transfert mobilities of track/soil system. A large amount of sites in Brussels were analysed but no validation was performed due to the lack of field data related to the passing of trams in the tested locations. The objective of this article is to develop a similar but fast approach using 2.5D coupled boundary element/finite element (BE-FE) model for the track/soil, dedicated to the study of localized defects, instead of experimental mobilities. A validation is proposed using experimental data and results obtained from a full 3D FE model.

2. Numerical model

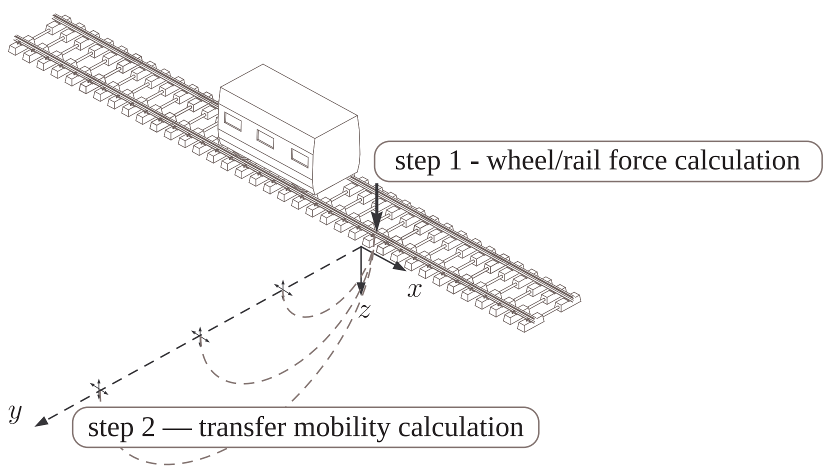


Figure 2: Modelling approach for ground vibration from local sources of excitation.

Based on the aforementioned statements, the problem can be split into two calculations (Figure 2):

- The wheel/rail force acting at the localized defect is calculated using a vehicle/track model. Including the track and the foundation in the simulation takes into account the track flexibility at the contact point, necessary for an accurate value of the wheel/rail force [11].
- The track/soil transfer function is calculated from a 2.5D coupled BE-FE model. This offers a fast way to obtain the dynamic interaction between a railway track and the underlying soil.

Each of these calculation steps are explained in the following subsections.

2.1 The vehicle/track system

The vehicle is considered as an assembly of rigid bodies, so as the multibody system approach can be systematically applied (Figure 3(a)). By fixing the generalized coordinates q_j ($j = 1, \ldots, n_{cp}$, n_{cp} being the number of degrees of freedom) defining the motion of each body and applying the generalized coordinate approach, a system of n_{cp} pure differential equations are built using the virtual power principle

$$\sum_{i=1}^{n_B} \left[\underline{\mathbf{d}}^{i,j} \cdot (\underline{\mathbf{R}}_i - m_i \underline{\mathbf{a}}_i) + \underline{\boldsymbol{\theta}}^{i,j} \cdot (\underline{\mathbf{M}}_{Gi} - \Phi_{G_i} \underline{\dot{\boldsymbol{\omega}}}_i - \underline{\boldsymbol{\omega}}_i \times \Phi_{G_i} \underline{\boldsymbol{\omega}}_i) \right] = 0 \, . \tag{1}$$

For each of the n_B bodies of the vehicle, m_i and Φ_{G_i} are the corresponding mass and central inertia tensor, $\underline{\mathbf{R}}_i$ and $\underline{\mathbf{M}}_{Gi}$ are the resultant force and moment. $\underline{\mathbf{a}}_i$ is the acceleration of the centre of gravity, $\underline{\mathbf{d}}^{i,j}$ the partial contributions of $\dot{q}_j$ in the velocity $\underline{\mathbf{v}}_i$ defined by

$$\underline{\mathbf{v}}_i = \sum_{j=1}^{n_{cp}} \underline{\mathbf{d}}^{i,j} \cdot \dot{q}_j \tag{2}$$

and $\underline{\boldsymbol{\theta}}^{i,j}$ the partial contributions of $\dot{q}_j$ in the rotational velocity $\underline{\boldsymbol{\omega}}_i$ defined by

$$\underline{\boldsymbol{\omega}}_i = \sum_{j=1}^{n_{cp}} \underline{\boldsymbol{\theta}}^{i,j} \cdot \dot{q}_j \, . \tag{3}$$

The position and orientation of each body (car body, bogie, wheelset, or other inertial component, with a role in the vehicle dynamics) is expressed by means of a homogeneous transformation matrix, which gives the situation of the associated local frame with respect to another frame in function of the configuration parameters q_j. This 4×4 matrix has the general well–known form

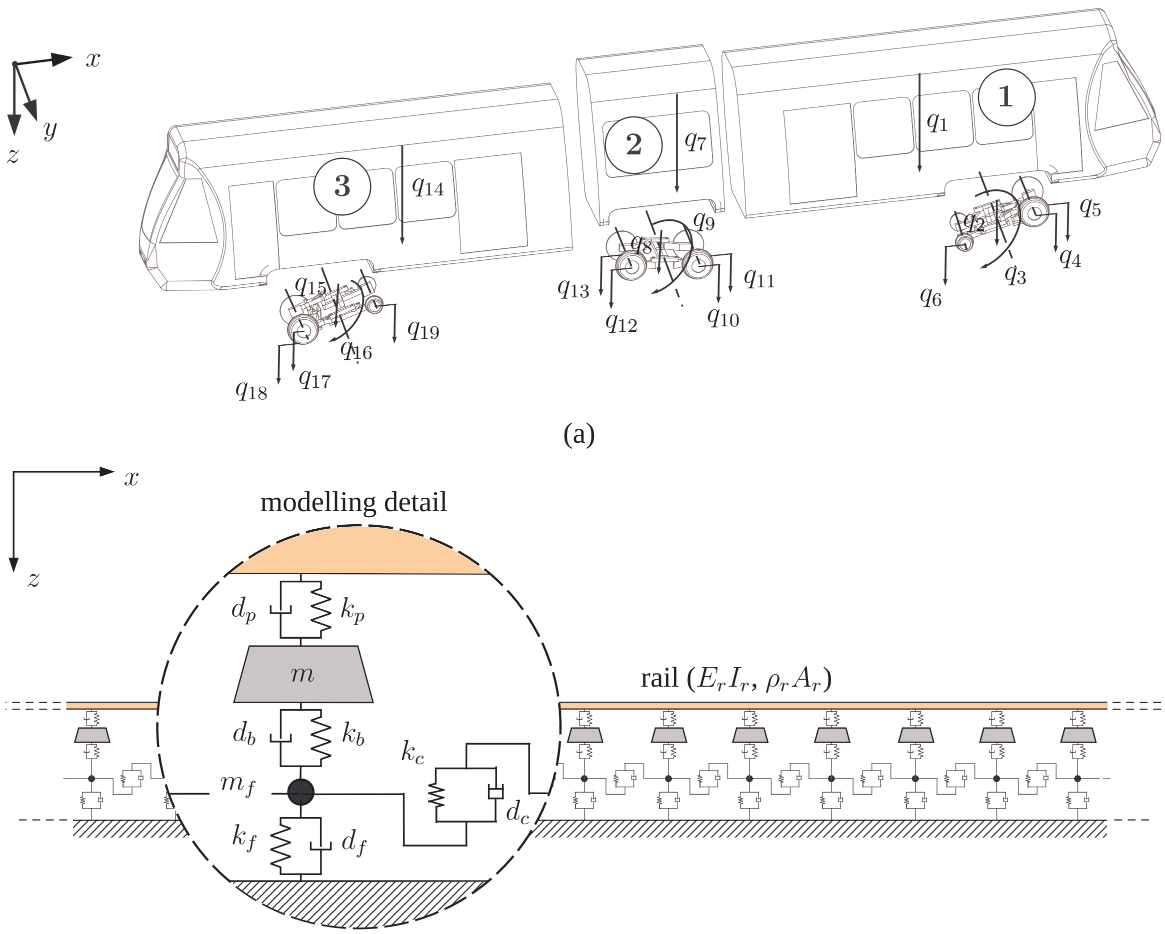
$$\mathbf{T}_{i,j} = \begin{pmatrix} \mathbf{R}_{i,j} & \{\underline{\mathbf{r}}_{j/i}\}_i \\ 0\,0\,0 & 1 \end{pmatrix} \tag{4}$$

where $\underline{\mathbf{r}}_{j/i}$ is the coordinate vector of frame j with respect to frame i, and $\mathbf{R}_{i,j}$ is the rotation tensor describing the orientation of frame j with respect to frame i. For any complex mechanical system, the motion of each body can be decomposed into a succession of elementary motions defined as successive multiplications of simpler matrices. To obtain the kinematics ($\underline{\mathbf{v}}_i$, $\underline{\mathbf{a}}_i$, $\underline{\boldsymbol{\omega}}_i$, $\underline{\dot{\boldsymbol{\omega}}}_i$) and the partial contributions ($\underline{\mathbf{d}}^{i,j}$, $\underline{\boldsymbol{\theta}}^{i,j}$), only differentiation operations on terms of homogeneous transformation matrices are necessary. Suspensions, as well as wheel/rail contacts, are defined as non-linear force elements applied to specific locations of bodies.

The track consists of a 2D two-layer finite element/lumped mass model (Fig. 3(b)). The rail (bending stiffness $E_\mathrm{r} I_\mathrm{r}$, mass per unit length $\rho_\mathrm{r} A_\mathrm{r}$) is described by finite elements (Euler-Bernoulli beams), discretely supported by the sleepers of half-mass m. A regular spacing L has been considered. Rail pads and ballast are characterised by springs and dampers (k_p and d_p for the rail pad, k_b and d_b for the ballast). A 2D model for the track is sufficient since the major contribution of ground vibration is induced by the vertical track deflection. The foundation is taken into account by means of a coupled lumped mass model [12] which consists of:

- several foundation elements of value m_f representing a fictitious mass of soil below the ballast/soil area moving as a rigid body,
- spring and damper elements (k_f, d_f) representing the direct foundation stiffness,
- spring and damper elements (k_c, d_c) representing the coupling foundation properties.

All these parameters are obtained by fitting model receptances with numerical or experimental counterparts [12].



(a)

(b)

Figure 3: Vehicle/track modelling: (a) vehicle kinematic configuration and (b) track model.

The proposed method was implemented in a C++ object-oriented program, using the in-house EasyDyn library [13]. An application based on the Python platform generates symbolic kinematic expressions. It creates a C++ code directly compilable against the EasyDyn library which is completed by the applied forces (suspensions, wheel/rail contact) and the link with the track model (already established and only depending on the site parameters). Each wheel/rail contact $i : 1 \mapsto n_w$ (n_w being the number of wheels in contact with the defect) is defined using the non-linear Hertz theory

$$F_{\text{wheel/rail},i} \quad = \quad K_{Hz} \left(z_{\text{wheel,i}} - z_{\text{rail}}(x_j) - h_{\text{defect}} \right)^{3/2} . \tag{5}$$

where $z_{\text{wheel,i}}$ is the vertical position of the wheel i and $z_{\text{rail}}(x_j)$ the corresponding vertical displacement at the rail at coordinate x_j. K_{Hz} is the Hertz's coefficient and h_{defect} the geometry of the imposed defect.

2.2 The track/soil system

The track-soil system is represented by a 2.5D BE-FE model (Figure 4). In this ballasted track model, the rails are also represented by Euler-Bernoulli beams with a bending stiffness $E_{\mathrm{r}}I_{\mathrm{r}}$ and a mass $\rho_{\mathrm{r}}A_{\mathrm{r}}$ per unit length. The rail displacements are denoted as $u_{\mathrm{r}1}(x_1, t)$ and $u_{\mathrm{r}2}(x_2, t)$. The positions of the rail are determined by x_1 and x_2, with $x_2 - x_1$ equal to the track gauge w_{r}. The internal energy dissipation in the rail is modelled by a loss factor η_{r}. The rail pads are modelled as continuous spring-damper connections. The rail pad stiffness k_{rp} and damping coefficient c_{rp} of a single rail pad are used to calculate the equivalent stiffness $\bar{k}_{\mathrm{rp}} = k_{\mathrm{rp}}/d$ and damping $\bar{c}_{\mathrm{rp}} = c_{\mathrm{rp}}/d$ being d the sleeper spacing. The ballast bed is represented by a set of distributed linear springs and dampers. The smeared ballast stiffness $\bar{k}_{\mathrm{b}}$ is computed from the vertical spring stiffness k_{b} per sleeper as k_{b}/d. The viscous damping in the ballast bed is accounted for by a ballast impedance equals $\bar{k}_{\mathrm{b}} + i\omega\bar{c}_{\mathrm{b}}$.

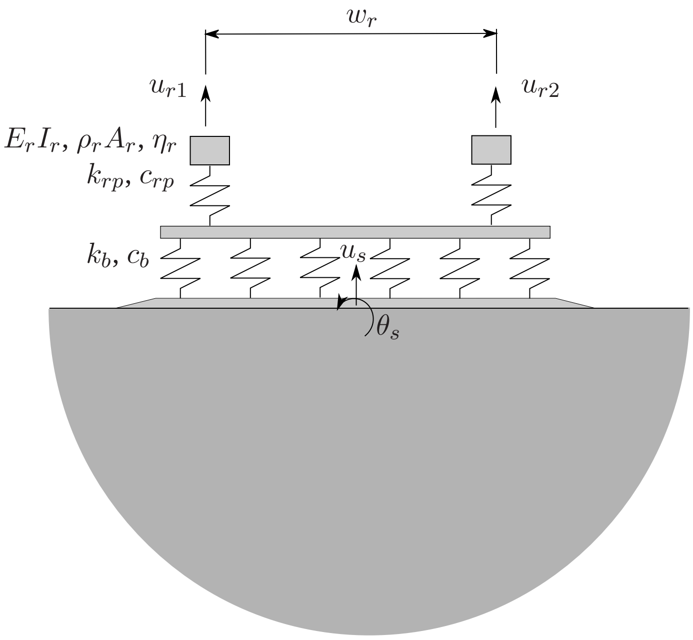


Figure 4: Cross section of ballasted track model.

The underlying soil is represented by means of a 2.5D boundary element method [14]. The fundamental solutions are computed with the direct stiffness method for a homogeneous or layered half-space [15]. The 2.5D FEM formulation can be elaborated as follows [16]:

$$\left[-\omega^2\mathbf{M}_{\mathrm{bb}} + \mathbf{K}^0_{\mathrm{bb}} + k_y^4\mathbf{K}^4_{\mathrm{bb}} + \tilde{\mathbf{K}}^{\mathrm{s}}_{\mathrm{bb}}(k_y,\omega)\right]\underline{\tilde{\mathbf{u}}}_{\mathrm{b}}(k_y,\omega) = \underline{\tilde{\mathbf{f}}}_{\mathrm{b}}(k_y,\omega) \tag{6}$$

where $\mathbf{K}^0_{\mathrm{bb}}$ and $\mathbf{K}^4_{\mathrm{bb}}$ are the stiffness matrices, $\mathbf{M}_{\mathrm{bb}}$ is the mass matrix, $\underline{\tilde{\mathbf{f}}}_{\mathrm{b}}(k_y,\omega)$ is the external load vector (including a unit force at the rail nodes), and $\mathbf{K}^{\mathrm{s}}_{\mathrm{bb}}(k_y,\omega)$ represents the dynamic soil stiffness matrix. A tilde above a variable denotes its representation in the frequency-wavenumber domain. The finite element matrices $\mathbf{M}_{\mathrm{bb}}$ and $\mathbf{K}^0_{\mathrm{bb}}$ to $\mathbf{K}^4_{\mathrm{bb}}$ in Eq. (6) are independent of the wavenumber k_y and the frequency ω and are only assembled once. Eq. (6) is now further elaborated by dividing the finite element degrees of freedom $\underline{\tilde{\mathbf{u}}}_{\mathrm{b}}(k_y,\omega)$ into internal degrees of freedom $\underline{\tilde{\mathbf{u}}}_{\mathrm{b}_1}(k_y,\omega)$ and degrees of freedom $\underline{\tilde{\mathbf{u}}}_{\mathrm{b}_2}(k_y,\omega)$ on the soil/structure interface:

$$\left(-\omega^2\begin{bmatrix}\mathbf{M}_{\mathrm{b}_1\mathrm{b}_1} & \mathbf{M}_{\mathrm{b}_1\mathrm{b}_2}\\ \mathbf{M}_{\mathrm{b}_2\mathrm{b}_1} & \mathbf{M}_{\mathrm{b}_2\mathrm{b}_2}\end{bmatrix}+\begin{bmatrix}\mathbf{K}^0_{\mathrm{b}_1\mathrm{b}_1} & \mathbf{K}^0_{\mathrm{b}_1\mathrm{b}_2}\\ \mathbf{K}^0_{\mathrm{b}_2\mathrm{b}_1} & \mathbf{K}^0_{\mathrm{b}_2\mathrm{b}_2}\end{bmatrix}+ k_y^4\begin{bmatrix}\mathbf{K}^4_{\mathrm{b}_1\mathrm{b}_1} & \mathbf{K}^4_{\mathrm{b}_1\mathrm{b}_2}\\ \mathbf{K}^4_{\mathrm{b}_2\mathrm{b}_1} & \mathbf{K}^4_{\mathrm{b}_2\mathrm{b}_2}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \tilde{\mathbf{K}}^{\mathrm{s}}_{\mathrm{b}_2\mathrm{b}_2}(k_y,\omega)\end{bmatrix}\right)\begin{bmatrix}\underline{\tilde{\mathbf{u}}}_{\mathrm{b}_1}(k_y,\omega)\\ \underline{\tilde{\mathbf{u}}}_{\mathrm{b}_2}(k_y,\omega)\end{bmatrix}=\begin{bmatrix}\underline{\tilde{\mathbf{f}}}_{\mathrm{b}_1}(k_y,\omega)\\ \underline{\tilde{\mathbf{f}}}_{\mathrm{b}_2}(k_y,\omega)\end{bmatrix} \quad (7)$$

Once the equilibrium equation (7) for the dynamic soil-track interaction problem has been solved, the integral representation theorem is applied to compute the radiated wave field from the tractions $\underline{\tilde{\mathbf{t}}}_{\mathrm{s}}(k_y,\omega)$ and displacements $\underline{\tilde{\mathbf{u}}}_{\mathrm{s}}(k_y,\omega)$ at the soil-structure interface:

$$\underline{\tilde{\mathbf{u}}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)=\tilde{\mathbf{U}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)\,\underline{\tilde{\mathbf{t}}}_{\mathrm{s}}(k_y,\omega)-\tilde{\mathbf{T}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)\,\underline{\tilde{\mathbf{u}}}_{\mathrm{s}}(k_y,\omega) \quad (8)$$

where the matrices $\tilde{\mathbf{U}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)$ and $\tilde{\mathbf{T}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)$ follow from the introduction of the boundary element discretization in the integral representation theorem and the vector $\underline{\tilde{\mathbf{u}}}_{\mathrm{r}}(\mathbf{x},k_y,\omega)$ collects the displacement components at n_{f} receiver locations. A discrete Fourier transform is then applied to obtain the transfer mobility functions at these receiver locations. Coupled to the discrete Fourier transform of Eq. (5), the displacement is finally obtained. An inverse Fourier transform then allows for time history representation.

3. Case study

The T2008 tram is a medium-sized system running in Brussels, with several interesting peculiarities. The specific low-floor imposed a large unsprung mass design that typically generates large ground vibrations, especially at low speeds and when passing on localized defects [5, 9]. Figure 5 presents the studied configuration, composed of a small centre car surrounded by two large cars. Track and soil dynamics parameters are those presented in [5].

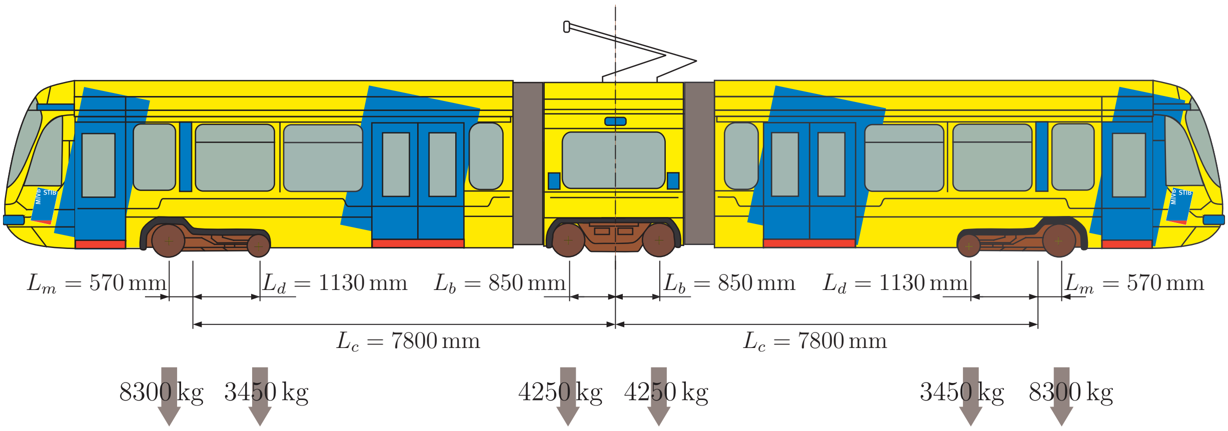


Figure 5: Main dimensions and axle loads of the T2008 tram.

Figure 6 shows the results from the proposed model (with the 2.5D approach) compared to those obtained from the full 3D model [5] and the corresponding measurement. The studied defect is a step discontinuity on the rail head and the receiver is located at $2\,\mathrm{m}$ from the track (from the external side of track, i.e. at $2.725\,\mathrm{m}$ from the track centerline). Peak particle velocity (PPV) and maximum transient vibration value ($MTVV$) indicators are included for a quantitative comparison. Figure 7 compares these results in term of one-third octave band spectra in the frequency range $1 - 100\,\mathrm{Hz}$. An overall good agreement is observed, validating the proposed approach. More particularly, the impact of each wheel on the local defect is clearly visible and only predicted by the proposed approach: the moving load effect is not taken into account in the modelling approach since only the dynamic interaction is calculated (this distinction is treated and explained in [17]).

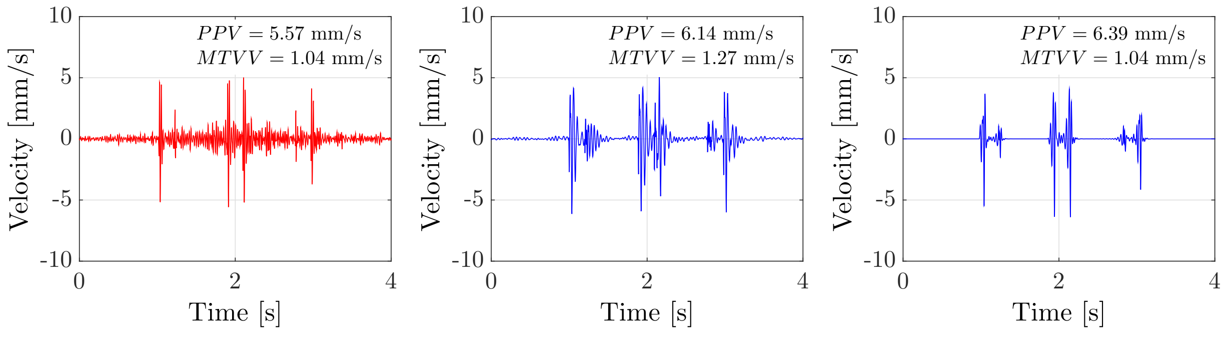


Figure 6: Time histories of vibration velocity at $2\,\mathrm{m}$ from the track for a tram speed of $30\,\mathrm{km/h}$: (a) experiments, (b) full 3D model and (c) proposed 2.5D approach.

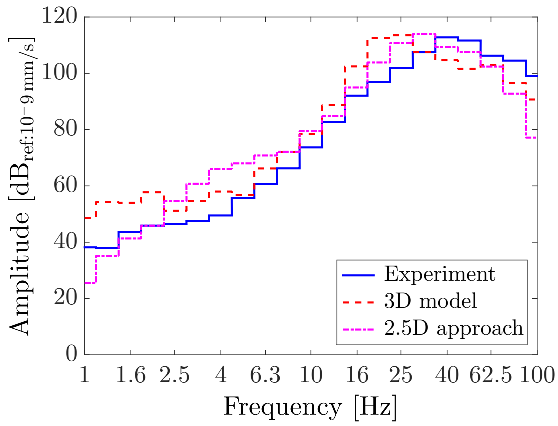


Figure 7: One-third octave band spectra of ground vibrations at $2\,\mathrm{m}$ from the track.

4. Conclusion

A fast computation approach was proposed in this article to evaluate the impact of localized defects on urban environments. A distinction between the static contribution (moving load) and the dynamic contribution (interaction between the vehicle and the defect) of the passing of a tram was made to establish a 2-step simulation taking into account the interaction of rail vehicles with a singular defect. The numerical results were validated using a combination of experiment data and data from a comprehensive prediction model. It was shown that accurate results can be obtained if attention is paid to the dynamic characteristics of localised defects.

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